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TRANSONIC FLOW PAST DUCTED BODIES OF REVOLUTION

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ABSTRACT. Application of exact particular solutions in the form of polynomials in r (where r is a cylindrical coordinate) of the equations describing supersonic gas flows to the construction of a ducted body of revolution situated in a low-supersonic gas flow. The results indicate that in the solutions, shock waves attenuate asymptotically at $r \to \infty$, as a result of which the solutions contain a rarefaction-wave envelope. (A67-27989)

For the purpose of investigating axial symmetric gas flows in a transonic velocity region, Karman introduced the dimensionless functions v_x', v_r' and the independent variables x', r', which are related to the particle velocity components v_x, v_r, density ρ, specific volume $V = 1/\rho$, pressure p, velocity of sound a, specific entropy s, and the cylindrical coordinates x, r by means of the following relations [1] /114*

$$\begin{gathered} v_x = a_*[1 + \varepsilon^{2/3}(2m_*)^{-1/3} v_x'], \qquad v_r = \varepsilon a_* v_r' \\ x = Lx', \qquad r = \varepsilon^{-1/3} L (2m_*)^{-1/3} r', \qquad m_* = (2\rho_*^3 a_*^2)^{-1}(\partial^2 p / \partial V^2)_s \end{gathered} \tag{1}$$

Here ε is a small parameter, L is a characteristic dimension, and the asterisk denotes the critical state of the gas. The new unknown functions v_x' and v_r' are found by solving a system of differential equations

$$-v_x \frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} = 0, \qquad \frac{\partial v_x}{\partial r} = \frac{\partial v_r}{\partial x} \tag{2}$$

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*Number in the margin indicates the pagination in the original foreign text.

where for brevity we omitted primes over all variables.

Reference [2] analyzes the particular solutions of the system of Equations (2) which are in the form of polynomials in r

$$v_x = f_2(\xi)r^2 + f_0(\xi), \quad v_r = g_3(\xi)r^3 + g_1(\xi)r, \quad x = \xi r^2 + h_0(\xi) \tag{3}$$

The functions of the parameter, appearing in (3), satisfy a system of ordinary differential equations

$$\frac{df_2}{d\xi} = -\frac{4(\xi f_2 - g_3)}{f_2 - 4\xi^2}, \quad \frac{dg_3}{d\xi} = \frac{2(f_2^2 - 4\xi g_3)}{f_2 - 4\xi^2}$$
$$\frac{df_0}{d\xi} = \frac{2g_1}{f_2 - 4\xi^2}, \quad \frac{dg_1}{d\xi} = \frac{2(f_2 f_0 - 2\xi g_1)}{f_2 - 4\xi^2}, \quad \frac{dh_0}{d\xi} = \frac{f_0}{f_2 - 4\xi^2} \tag{4}$$

The first two equations in (4) can be separated from the rest. Their properties were investigated in [3], where they were used to construct the velocity fields in Laval nozzles of circular cross-section. Solutions of the form of (3) were first obtained in [4] for the equations of short waves, which describe flows with small but sharp variations of the gas parameters in narrow regions adjoining shock wave fronts.

The particular solutions (3) result in a flow with a parabolic shock wave

$$x = \xi_2 r^2 + h_0(\xi_2), \quad \xi_2 = \text{const}$$

at the front of which two boundary conditions must be satisfied. The first of them is the equation of Busemann's shock polar [5]; the second consists of the continuity condition on the velocity component which is tangential to the shock front. These boundary conditions result in the Cauchy problem [2]

$$f_2(\xi_2) = 8\xi_2^2, \quad g_3(\xi_2) = -16\xi_2^3, \quad f_0(\xi_2) = -v_{x\infty}, \quad g_1(\xi_2) = 4v_{x\infty}\xi_2 \tag{5}$$

for the functions involved in the system of ordinary differential equations (4). The constant $v_{x\,\infty} > 0$ is determined by the velocity of the oncoming uniform supersonic flow, the value of $h_0(\xi_2)$ specifies the position of the shock.

Let us now construct by means of the solution (3) a ducted body of revolution immersed in a gas moving at a low supersonic velocity. As the characteristic dimension L in Equations (1), we shall take the radius of the duct, and as the small parameter ε we shall take the angle θ_0 at the vertex, which will be considered so small that the shock wave is attached. The origin of the coordinates will be placed on the axis of symmetry of the flow, so that the position x = 0 will correspond to the leading edge of the body over which the flow occurs. Hence, we find

$$h_0(\xi_2) = -\theta_0 \text{[illegible]} (2m_*) \text{[illegible]} \xi_2 \tag{6}$$

The solution of the Cauchy problem (5), (6) for the system (4) gives the desired velocity field. The shape of the contour r = R(x) of the body over which the flow occurs in terms of the initial variables is given by the equations

$$R = r \left\{ 1 + \text{[illegible]} (2m_*) \text{[illegible]} \int [\theta_0 \text{[illegible]} (2m_*) \text{[illegible]} + g_0(\xi)] \left[\theta_0 \text{[illegible]} (2m_*) \text{[illegible]} + \frac{f_0(\xi)}{h_0(\xi) - \xi^2} \right] d\xi \right\}$$
$$x = \text{[illegible]} + h_0(\xi)] \tag{7}$$ /115

We must express the parameter ξ_2 in terms of $v_{x\,\infty}$ and θ_0. For this purpose, it is simplest to use the condition that the velocity behind the shock wave at the leading edge of the body be directed along its contour. Using Equation (1) and the initial conditions (5), we obtain for ξ_2 a cubic equation

$$\xi_2^3 - \frac{v_{x\infty}}{4\theta_0^{3/2}(2m_*)^{\text{[illegible]}}}\xi_2 + \frac{1}{32\theta_0 m_*} = 0$$

As the solution of the latter, we take

$$\xi_0 = \frac{v_{x\infty}^{1/2}}{3^{1/2}\theta_0^{1/3}(2m_*)^{1/3}} \cos\frac{\pi - \varphi_0}{3}, \quad \cos\varphi_0 = \frac{3^{3/2}}{4 v_{x\infty}}$$

which corresponds to the weaker of the two possible shock waves which are formed in a flow over an infinite wedge with given half-angle θ_0 at the vertex.

One of the fields of flow is shown in Figure 1, where we plotted the lines of constant velocity $v_x(x, r) = \text{const}$, the shock wave and the characteristics inclined in the direction of the gas flow C_+. Figure 2 shows the body of revolution which perturbs the flow. Its contour is defined by Formulas (7). The Mach number M_∞ of the oncoming flow in the calculations was taken as 1.2 ($v_{x\,\infty} = 1.229$), the edge angle of the leading edge of the body was $\theta_0 = 0.072$, and the coefficient $m^* = 1.2$ (which corresponds to the ideal gas whose Poisson adiabatic exponent was $x = 1.4$). With these values of the parameters, the velocity at the body directly behind the shock wave was subsonic, and then it gradually rose to supersonic.

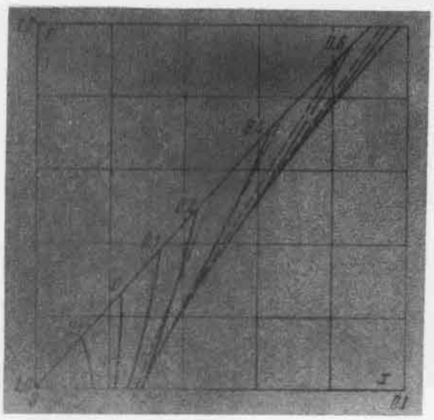

Figure 1

The most interesting feature of the flows under consideration is the occurrence of the limiting line in the supersonic region of the velocity field (Figure 1). Its appearance is

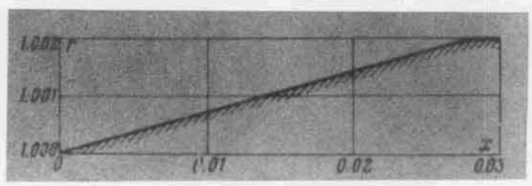

Figure 2

due to the fact that, along the straight lines r = const, the coordinate $x(\xi)$ attains a maximum. In a parametric form the limiting line is determined by the formulas

$$x_*(\xi) = \frac{\xi f_0(\xi)}{f_1 - 4\xi^2}, \quad r_* = \left(-\frac{f_0(\xi)}{f_1 - 4\xi^2}\right)^{1/2} \tag{8}$$

At those points of the curve (8), the derivatives of the velocity vector components with respect to the coordinates become infinite, and the radius of curvature of the streamlines is zero. The limiting line is the simultaneous envelope of the curves $v_x(x, r)$ = const and the rarefaction waves which merge with it and come to an end at the shock wave. This line begins on the body at the point where its contour has a cuspidal point, and is propagated below the flow, reaching the shock front tangent to the latter. At the point of contact between the shock wave and the rarefaction wave envelope, it degenerates into the characteristic with zero excess pressure.

Simultaneously from the point of the intersection of the body contour with the limiting line, there is a C_+ characteristic slanted downwards along the flow, up to which the velocity field is specified by analytic functions. A further analytic continuation of the flow beyond the characteristic in question is impossible; it does not correspond to any real body of revolution. In addition, such a continuation would mean that in the supersonic flow with $M_\infty > 1$, the shock wave of parabolic form $\xi = \xi_2$ = const, should degenerate into a characteristic at a finite distance from its point of appearance. But, as we know [6], the attenuation of the shock waves occurs asymptotically for $r \to \infty$. This fact means that the solution (3) contains the envelope of the rarefaction waves; the fields of the plane parallel flows, investigated in detail in [2], must have a similar structure. /116

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